ZESZYTY NAUKOWE UNIWERSYTETU SZCZECIŃSKIEGO nr 855
**Finanse, Rynki Finansowe, Ubezpieczenia nr 74, t. 1 (2015)**

DOI: 10.18276/frfu.2015.74/1-19 s. 217–227

# Long-Term Relationships between the Stock Exchanges in Frankfurt, Vienna and Warsaw*

**Tomasz Wójtowicz****

**Abstract:** In this paper we have studied the existence and the nature of long-range relations between the main indices of three European stock markets: in Frankfurt, Vienna and Warsaw. The first two of them are developed markets, while the last one is seen as an emerging market. On the basis of daily data from the period 2003–2014 we analysed the $I(1)/I(0)$ co-integration of market indices. The results of this commonly applied technique are compared with the results of the more flexible fractional co-integration analysis.

**Keywords:** cointegration; fractional cointegration; stock markets; emerging markets

## Introduction

In the last few decades, globalization has led to increasing connections between equity markets in various parts of the world. These connections have caused a co-movement and correlation of stock prices. The existence of such connections is important, because it can lead to spillovers and contagion effects. The issue of the cointegration of European stock markets has been widely analysed in the literature. For example, Fratzcher (2002) indicates the increasing integration of equity markets in the Euro zone. This observation is also confirmed by Adam et al. (2002). On the other hand, the integration of emerging European equity markets is not so obvious. Capiello et al. (2006) indicate that the degree of integration with developed markets depends on the size and capitalization of a market. Similar results are reported by Yusupova (2005). Quite different results were obtained by Mrzygłód (2011). On the basis of monthly data she proves a weak cointegration between emerging European markets. Only Polish and Hungarian stock markets can be seen as cointegrated. Moreover, Mrzygłód (2011) does not confirm the existence of the cointegration between emerging and developed European stock markets. However, Czupryna (2013) indicates that the analysis of cointegration of stock markets is sensitive to the construction of market indices. Significant

* Financial support for this paper is from the National Science Centre of Poland (Research Grant DEC-2012/05/B/HS4/00810) is gratefully acknowledged.

** dr Tomasz Wójtowicz, Akademia Górniczo-Hutnicza im. Stanisława Staszica w Krakowie, Wydział Zarządzania, Al. Mickiewicza 30, 30-059 Kraków, e-mail: twojtow@agh.edu.pl.

cointegration of the Warsaw Stock Exchange and the Frankfurt Stock Exchange is observed when the WIG20 is replaced in the analysis by the WIG20TR.

In this paper we examine the cointegration between Austrian, German and Polish stock markets. These are very different stock markets. The Frankfurt Stock Exchange is one of the largest stock markets in Europe. Its capitalization is about ten times greater than the capitalization of stock markets in Vienna and Warsaw. The Vienna Stock Exchange, similarly to the FSE, is a developed market. However, its capitalization is similar to the capitalization of the Warsaw Stock Exchange. Both of them are among the largest stock markets in Central and Eastern Europe.

We have extended previous works about the relationships between European stock markets by applying the concept of fractional cointegration. This allows us to describe the relationships between the stock markets in Vienna, Frankfurt and Warsaw more accurately.

The rest of the paper is as follows. In the next section we briefly describe co-integration with integer orders. Section two contains basic information about fractional integration and co-integration. The data used in the analysis is described in Section three. Section four contains a presentation of the main empirical results. A short summary concludes the paper.

## 1. Cointegration

The concept of cointegration was initiated by Granger (1981). According to him, two time series $x_t$ and $y_t$ are cointegrated of order $(d, b)$ (denoted by $CI(d, b)$) if

1) $x_t$ and $y_t$ are integrated of order $d$ (are $I(d)$ processes);
2) there exists a constant $\beta$ such that process $\varepsilon_t = y_t - \beta x_t$ is integrated of order $d - b$ (is $I(d - b)$ process), with $b > 0$.

In practice, the most common is a cointegration of order (1, 1) when both processes $x_t$ and $y_t$ are unit-root nonstationary, while there exists their a stationary linear combination.

There are two main methods of testing for the existence of a cointegration between nonstationary processes: the Engle-Granger two-step procedure (Engle, Granger 1987) and Johansen cointegration tests (Johansen 1988; Johansen, Juselius 1990).

In the Engle-Granger procedure we first test the existence of the unit root in $x_t$ and $y_t$. This is generally made on the basis of an ADF test. Then, by the OLS, parameters of the model $y_t = \alpha + \beta x_t + \varepsilon_t$ are estimated. The final step is a unit-root test of the residuals $\varepsilon_t$. If the process $\varepsilon_t$ is stationary, then $x_t$ and $y_t$ are cointegrated with the cointegrating vector $[1, -\beta]$. Otherwise, the cointegrating relation does not exist.

The Johansen tests for a $p$-dimensinal vector time series $X_t$ are based on a VAR representation of $X_t$. This gives a general VAR model with $k$ lags:

$$X_t = A_1 X_{t-1} + \cdots + A_k X_{t-k} + \mu + \Phi \mathrm{D_t} + \varepsilon_\mathrm{t} \qquad (1)$$

the following VECM representation can be written:

$$\Delta X_t = \Pi X_{t-k} + \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_{k-1} \Delta X_{t-k+1} + \mu + \Phi D_t + \varepsilon_t \tag{2}$$

where $\Pi = \Pi_1 + \ldots + \Pi_k - I$ and $\Gamma_i = \Pi_1 - \ldots - \Pi_i - I$ for $I = 1, \ldots, k - 1$.

The rank $r$ of the matrix Π determines the existence of the cointegration between elements of $X_t$. It also determines the number of cointegrating relations. When $r = 0$, all the processes in $X_t$ are $I$ (1) and are not cointegrated. On the other hand, when $r = p$, then all the processes in $X_t$ are $I$ (0). Cointegrating relations between elements of $X_t$ exist only when $0 < r < p$.

The rank of Π can be determined by testing the significance of its eigenvalues. Let $\lambda_1, \ldots, \lambda_p$ are the eigenvalues of Π written in ascending order ($\lambda_1 \geq \ldots \geq \lambda_p$). The two Johansen tests can be applied to establish the rank of Π. These are the "λ – max" test and the "trace" test. The "λ – max" test verifies the null hypothesis of the significance of the individual $k - th$ eigenvelue and the hypotheses about the rank of Π are: $H_0$: $r = k$ and $H_1$: $r = k + 1$. The "trace" test verifies the hypothesis of the joint significance of the first $k$ eigenvalues and thus the hypotheses about the rank of Π have the form: $H_0$: $r = k$ and $H_1$: $r > k$. Johansen and Juselius (1990) provide critical values for the statistics in both tests.

## 2. Fractional integration and cointegration

In the analysis of the integration we do not have to restrict attention only to integer integration orders. We can also apply the concept of fractional integration which is more flexible.

A covariance stationary process is fractionally integrated with the integration order $d$ if its spectral density function $f(\lambda)$ satisfies:

$$f(\lambda) \sim c\lambda^{-2d} \quad \text{as} \quad \lambda \to 0^+ \tag{3}$$

where c is a finite positive constant and the symbol "~" means that the ratio of the left- and right-hand sides tends to be at the limit. When $d > 0$, the process exhibits a long memory and its autocorrelation function dies out at a hyperbolic rate (Granger, Joyeux 1980). The above definition holds for $d < 0.5$ when a process is stationary. However, it has been generalized also on non-stationary cases when $d < 0.5$. Fractionally integrated processes with the integration order $d$ fill the gap between stationary $I(0)$ processses and nonstationary $I(1)$ processes.

There are several methods of estimating the fractional integration order $d$ (called also a long memory parameter). The majority of them are semi-parametric methods based on log-periodogram regression (Geweke, Porter-Hudak 1983) or based on a modification of local Whittle methods (Shimotsu 2010). The advantage of semi-parametric methods is that they are based on the analysis of the spectral density function of the process for very low frequencies. Hence, these methods are robust for the short-term behaviour of the process described by higher frequencies. Semi-parametric methods are also free of any problems due to model misspecification.

The development of the concept of long memory and fractional integration (Robinson 2005; Shimotsu 2006, among others) has led to the extension of the cointegration definition to non-integer integration orders. We will consider the simplest case of the cointegration of two processes. Several definitions of fractional cointegration can be found in the literature (see for example Robinson, Marinucci 2001; Robinson, Yajima 2002). Because the previously mentioned definition of cointegration is not restricted to the integer orders of integration, the most common definition of fractional cointegration is as follows. Two fractionally integrated series $x_t$ and $y_t$ are cointegrated in order $d$ if:

1) $x_t$ and $y_t$ share the same long memory, i.e. $d = d_x = d_y = d$;
2) there exists a constant $\beta$ such that the process $\varepsilon_t = y_t - \beta x_t$ has a long memory parameter $d_\varepsilon$ such that $d_\varepsilon < d_x$.

From the above definition it follows that the existence of the common integration order $d$ of the two processes is a necessary condition for their fractional cointegration. When long memory parameters ($d_x$, $d_y$) are multivariate ELW estimators proposed by Shimotsu and Phillips (2005) then the null hypothesis $H_0$: $d_x = d_y$ can be tested by the statistic (Robinson, Yajima 2002)

$$T_{xy} = \frac{m^{0.5}(\hat{d}_x - \hat{d}_y)}{\sqrt{0.5\left(1 - \hat{G}_{xy}^2/(\hat{G}_{xx}\hat{G}_{yy})\right)} + h(T)} \tag{4}$$

where $\hat{d}_x$ and $\hat{d}_y$ are estimates of the fractional integration orders of $x_t$ and $y_t$, $\hat{G}_{xx}$, $\hat{G}_{yy}$, $\hat{G}_{xy}$ are diagonal and off-diagonal elements of the cross-spectral density matrix $\hat{G}$ at frequency zero, $T$ is the length of data, $h(T)$ and $m$ are tuning parameters such that for $\xi \in (0, 2]$:

$$\frac{1}{m} + \frac{m^{1+2\xi}(\ln m)^2}{T^{2\xi}} \to 0 \; as \; T \to \infty \tag{5}$$

and

$$h(T) + \frac{(\ln m)m^{0.5+\xi}/T^{\xi} + (\ln m)^2 m^{-1/6}}{h(T)} \to 0 \; as \; T \to \infty \tag{6}$$

Under some regularity assumptions (see Robinson, Yajima 2002) $T_{xy}$ the statistic has asymptotic standard normal distribution when $x_t$ and $y_t$ are not cointegrated. When $x_t$ and $y_t$ are cointegrated, then $T_{xy}$ tends to be zero.

The existence of the common long memory parameter is a necessary condition for fractional cointegration, but it is not a sufficient condition. The existence of fractional cointegrating relations can be analysed following two different semiparametric methodologies. The first (Shimotsu 2012) is similar to the two-step Engle-Granger procedure. The exact local Whittle estimation of the fractional cointegration of the two processes proposed by Shimotsu (2012) involves the joint estimation of the cointegrating vector [1, –β] and long memory parameters of $x_t$ and the regression residuals $\varepsilon_t$. The second method, proposed by Robinson and Yajima (2002) (and developed further by Nielsen and Shimotsu, 2007) for the analysis of cointegration in a $p$-dimensional random process, is similar to Johansen's procedure for *CI* (1, 1). It contains an estimation of a common long memory parameter $d^*$ of the time series under study. Then, the cross-spectral density matrix $\hat{G}(d^*)$ is estimated at frequency zero.[1] The rank $r$ of $\hat{G}(d^*)$ determines the number of fractional cointegration relationships between processes. Similarly to Johansen's procedure, the dermination of rank $r$ is based on the eigenvalues of $\hat{G}(d^*)$. Let $\lambda_1 \geq \ldots \geq \lambda_p$ are the eigenvalues of $\hat{G}(d^*)$, then the rank estimate is defined as

$$\hat{r} = \underset{u=0,\ \ldots,\ p-1}{\arg\min}\ L(u) \tag{7}$$

where

$$L(u) = v(T)(p-u) - \sum_{i=1}^{p-u} \lambda_i \tag{8}$$

Definition of $L(u)$ contains an additional tuning function $v(T)$ such that

$$v(T) + \frac{1}{m_1^{0.5} v(T)} \to 0\ as\ T \to \infty \tag{9}$$

The procedure of the determination of the rank of $\hat{G}(d^*)$ is sensitive to the value of the bandwidth parameter $v(T)$, thus we will apply a range of values of $v(T) = m_1{}^q$ with $q = 0.25$, 0.35, 0.45 as indicated by Nielsen and Shimotsu (2007). In the same way, determination of the fractional cointegration rank can be also performed on the basis of the correlation matrix

$$\hat{P}(d^*) = \hat{D}(d^*)^{-0.5}\hat{G}(d^*)\hat{D}(d^*)^{-0.5} \tag{10}$$

where $\hat{D}(d^*) = diag\{\hat{G}_{11}(d^*), \ldots, \hat{G}_{pp}(d^*)\}$.

Described above the semiparametric methods of the analysis of fractional integration and cointegration are robust to short-term properties of the analysed processes. However, they require the application of user-chosen parameters, such as $m$, $m_1$, $h(T)$ and $v(T)$. Hence, to be more confident in the interpretation of the results, we performed an analysis for various appropriate values of these parameters.

[1] To ensure faster convergence, a new bandwidth parameter $m_1$ is used in the estimation of $\hat{G}(d^*)$.

## 3. Data

The analysis in this paper concentrates on the dependence of Austrian, German and Polish stock markets. Each market under study is represented by its main index, namely: ATX (for the Vienna Stock Exchange), DAX (for the Frankfurt Stock Exchange) and the WIG20 (for the Warsaw Stock Exchange). Originally, the WIG20 is denominated in PLN, while both ATX and DAX are denominated in euros. To reduce the impact of exchange rates, the WIG20 is also converted to euros. The analysis in this paper is based on natural logarithms of daily closing prices of the indices under study in the period from 2 January 2003 to 31 December 2014. The period under study contains different market phases, particularly; it covers the period of the global financial crisis. Due to nonsynchronous trading caused by different holidays, the samples under study had different lengths. To provide a time series of the same length, the missing data were replaced by the last closing price before the gap. As a result, we obtained the samples of 3,056 logarithms of daily closing prices of the ATX, DAX and WIG20.

## 4. Cointegration – empirical results

The necessary condition to consider *CI* (1, 1) cointegration between the time series is their nonstationarity. To test it we applied the commonly used ADF test.

**Table 1**

Results of the ADF tests for ATX, DAX and WIG20

| | Regression with no intercept nor time trend | | Regression with an intercept | | Regression with an intercept and a time trend | |
|---|---|---|---|---|---|---|
| | ADF statistic | p-value | ADF statistic | p-value | ADF statistic | p-value |
| ATX | 0.60 | 0.81 | –2.26 | 0.22 | –2.16 | 0.51 |
| DAX | 1.44 | 0.96 | –1.49 | 0.50 | –2.27 | 0.46 |
| WIG20 | 0.45 | 0.76 | –2.18 | 0.24 | –1.94 | 0.60 |

Source: author's own computation.

The results of the performed tests with different hypothesis are presented in Table 1. Subsequent columns contain the values of the test statistics and p-values for the ADF test without constant, with a constant and with a linear time trend. The null hypothesis about the existence of the unit root cannot be rejected in any of the cases. Thus, all of the three indices under study are nonstationary and the existence of cointegrating relations between them can be analysed.

The existence of the pairwise cointegration between the indices under study was examined by both the Engle-Granger and Johansen procedures. Table 2 presents the values of statistics in trace tests and $\lambda$ – max tests for each pair of the indices. Because in the each case we consider a pair of time series, then only the existence of zero ($r$ = 0) or one ($r$ = 1) cointegrating relationship is examined. The three last columns contain critical values for each test. A comparison of the test statistics and the critical values indicates that only the ATX and WIG20 can be seen as being cointegrated – the null hypothesis that there is no cointegration between them is rejected by both tests at least at a 5% level. The results in Table 2 indicate that the two other pairs (ATX – DAX and DAX – WIG20) are not cointegrated *CI* (1, 1). These results are also confirmed by the two-step Engle-Granger procedure where only residuals from the regression of the WIG20 on the ATX are stationary.[2]

**Table 2**

Results of the Johansen cointegration tests

| | $r$ | ATX – DAX | ATX – WIG20 | DAX – WIG20 | Critical values | | |
|---|---|---|---|---|---|---|---|
| | | | | | 10% | 5% | 1% |
| Trace test with a constant | 1 | 2.61 | 6.39 | 1.92 | 7.52 | 9.24 | 12.97 |
| | 0 | 11.65 | $24.86^{***}$ | 7.88 | 17.85 | 19.96 | 24.6 |
| $\lambda$ – max test with a constant | 1 | 2.61 | 6.39 | 1.92 | 7.52 | 9.24 | 12.97 |
| | 0 | 9.03 | $18.47^{**}$ | 5.96 | 13.75 | 15.67 | 20.2 |

$^{*}, ^{**}, ^{***}$ – denote significance at a 10%, 5% and 1% level, respectively.

Source: author's own computation.

We begin the analysis of fractional cointegration between the ATX, DAX and WIG20 with the estimation of individual long memory parameters. Table 3 presents values of the Exact Local Whittle estimators (Shimotsu and Phillips 2005) computed for various values of bandwidth parameter $m$ satisfying (5).[3] All of the estimates in table 3 are close to 1, which is in line with the common view of index prices as a unit root process. However, this fact should be formally tested. Shimitsu and Phillips (2005) proved that the ELW estimator has asymptotically a normal distribution with a standard deviation $1/2\sqrt{m}$ . For the bandwidth $m = T^{0.65}$ the standard deviation of the ELW estimator is equal to 0.037. It means that the fractional difference parameters of the ATX, DAX and WIG20 are insignificantly different from one another.

**Table 3**

The ELW estimates of long memory parameters

| $m$ | ATX | DAX | WIG20 |
|---|---|---|---|
| $T^{0.35}$ | 1.083 | 1.040 | 1.098 |
| $T^{0.45}$ | 1.219 | 0.992 | 1.081 |
| $T^{0.55}$ | 1.177 | 1.046 | 1.036 |
| $T^{0.65}$ | **1.049** | **0.993** | **0.989** |
| $T^{0.75}$ | 1.008 | 0.956 | 0.986 |

Source: author's own computation.

[2] The ADF test rejects the null hypothesis at a 1% level.

[3] In the literature it is suggested to use $m = T^{0.65}$.

The necessary condition for the fractional cointegration of a two time series is the equality of their integration parameters. Table 4 presents the results of a pairwise test of the existence of common fractional integration orders for the indices under study. The tests are based on $T_{xy}$ statistics given by (4). In order to take into account the possible cointegration between the analysed time series, the columns in Table 4 present the values of $T_{xy}$ statistics computed for various $h(T)$. The value of $T_{xy}$ may by very sensitive to the choice of $h(T)$, however, when the null hypothesis is not rejected for small $h(T)$, it strongly suggests the equality of the integration orders. In each case reported in Table 4, the $T_{xy}$ statistic does not exceed critical values for a standard normal distribution. Hence, we can assume the equality of the integration orders in each pair of the indices under study. It indicates the possibility of the existence of the cointegration between the ATX, DAX and WIG20.

**Table 4**

Values of $T_{xy}$ statistics in the test of the equality of integration orders

| | $h(T) = 1 : (\sqrt{\ln T})$ | $h(T) = 1 : (\ln T)$ | $h(T) = 1 : (\ln^2 T)$ |
|---|---|---|---|
| ATX – DAX | 0.318 | 0.155 | 0.074 |
| ATX – WIG20 | 0.286 | 0.129 | 0.057 |
| DAX – WIG20 | 0.947 | 0.927 | 0.912 |

Source: author's own computation.

We first analysed the fractional cointegration in each pair of the indices by the exact local Whittle estimator of fractional cointegration proposed by Shimotsu (2012). For each pair of the indices, we estimated a common fractional integration order $d_x$ of both time series, the parameter $\beta$ in the regression $y_t = \beta x_t + \varepsilon_t$ and the fractional integration order $d_\varepsilon$ of the residual series $\varepsilon_t$. Additionally, to prove the significance of the cointegration, we tested the equality of the integration orders $d_x$ and $d_\varepsilon$. The relation is significant when $d_\varepsilon$ is significantly smaller than $d_x$. The results of this analysis can be seen in Table 5. In the case of all the pairs regression residuals have a significantly lower integration order than market indices. However, the significance levels are different. The difference for ATX and WIG20 is significant at a 1% level, for ATX and DAX it is significant at a 5% level, while the difference between $d_x$ and $d_e$ for DAX and WIG20 is significant at a 10% level. Hence, each pair

**Table 5**

Results of the exact local Whittle estimation of fractional cointegration

| | $d_x$ | $d_\varepsilon$ | $\beta$ | $T_{x\varepsilon}$ *statistic* | *p-value* |
|---|---|---|---|---|---|
| ATX – DAX | 1.05 | 0.96 | 0.39 | 2.26 | 0.0120 |
| ATX – WIG20 | 1.05 | 0.89 | 0.91 | 3.14 | 0.0009 |
| DAX – WIG20 | 0.99 | 0.91 | 0.47 | 1.60 | 0.0545 |

Source: author's own computation.

of the indices is fractionally cointegrated and this relation is the most pronounced for the ATX and WIG20 where the linear combination of the indices has the integration order (0.89) which is also a significantly smaller than one. This observation is in line with the results of the Johansen tests presented earlier.

In order to confirm the above results we additionally analysed the rank of cross-spectral density matrices $\hat{G}(d^*)$ and the correlation matrices $\hat{P}(d^*)$. For each pair, the procedure of rank determination contains the estimation of the common fractional integration order $d^*$ and the matrices $\hat{G}(d^*), \hat{P}(d^*)$. This estimation was performed with the new bandwidth parameter $m_1$ which must be smaller than the previously applied $m$. We considered $m_1 = T^{0.6}$. The number of cointegrating relations $r$ is computed from (7) and (8). In the definition of the function $L$ we applied $v(T)$ equal to $m_1^{-0.25}$, $m_1^{-0.35}$ and $m_1^{-0.45}$. For each pair of the indices under study, for different values of $v(T)$, Table 6 shows the values of the function $L$ and the estimated number $r$ of cointegrating relations. The 3 results computed on the basis of the cross-spectral density matrix $\hat{G}(d^*)$ are shown in Panel A, while the results based on $\hat{P}(d^*)$ are in Panel B. The rank analysis, particularly based on matrix $\hat{G}(d^*)$, supports the existence of fractional cointegration between the ATX, DAX and WIG20. For each pair and for each of the considered $v(T)$ in panel A there exists one cointegrating relation. The results for the correlation

**Table 6**

Results of the rank determination procedure

| | $v(T)$ | $m_1^{-0.25}$ | $m_1^{-0.35}$ | $m_1^{-0.45}$ |
|---|---|---|---|---|
| Panel A: analysis based on the cross-spectral density matrix $\hat{G}(d^*)$ | | | | |
| | $L(0)$ | 0.60 | 0.37 | 0.23 |
| ATX – DAX | $L(1)$ | 0.30 | 0.19 | 0.11 |
| | $r$ | 1 | 1 | 1 |
| | $L(0)$ | 0.60 | 0.37 | 0.23 |
| ATX – WIG20 | $L(1)$ | 0.30 | 0.19 | 0.11 |
| | $r$ | 1 | 1 | 1 |
| | $L(0)$ | 0.60 | 0.37 | 0.23 |
| DAX – WIG20 | $L(1)$ | 0.30 | 0.19 | 0.11 |
| | $r$ | 1 | 1 | 1 |
| Panel B: analysis based on the correlation matrix $\hat{P}(d^*)$. | | | | |
| | $L(0)$ | –1.40 | –1.63 | –1.77 |
| ATX – DAX | $L(1)$ | –1.53 | –1.65 | –1.72 |
| | $r$ | 1 | 1 | 0 |
| | $L(0)$ | –1.40 | –1.63 | –1.77 |
| ATX – WIG20 | $L(1)$ | –1.51 | –1.62 | –1.69 |
| | $r$ | 1 | 0 | 0 |
| | $L(0)$ | –1.40 | –1.63 | –1.77 |
| DAX – WIG20 | $L(1)$ | –1.43 | –1.54 | –1.62 |
| | $r$ | 1 | 0 | 0 |

Source: author's own computation.

matrix, however, depend on the $v(T)$. When $v(T)$ is large, one cointegrating relation is also indicated.

## Summary

In this paper we have shown the long-term relationships between prices of the ATX, DAX and WIG20 – the main indices of the stock exchanges in Vienna, Frankfurt and Warsaw. The analysis of the cointegration between these stock markets was performed on the basis of daily closing prices from 2 January 2003 to 31 December 2014. A two-step Engle-Granger procedure and Johansen tests confirmed the existence of a cointegrating relation between the ATX and WIG20. It indicates the existence of a strong long-term relationship between these major stock markets in Central and Eastern Europe. On the other hand, the ATX and WIG20 show no cointegration with the DAX.

The cointegration definition, however, is not restricted to integer integration orders. When we allow non-integer (fractional) integration orders, then additional long-term relations between the stock markets under study were revealed. The analysis of fractional cointegration by methods used by Shimotsu (2012) and Nielsen and Shimotsu (2007) confirmed that the indices under study were all pairwise and fractionally cointegrated. However, the strongest relationship exists between the ATX and the WIG20. It shows that the application of the fractional cointegration analysis allows for a more adequate description of the long-term relationships between stock markets.

## References

Adam K., Jappelli T., Menichini A., Padula M., Pagano M. (2002), *Analyse, Compare, and Apply Alternative Indicators and Monitoring Methodologies to Measure the Evolution of Capital Markets Integration in the European Union*, European Commission, Internal Market.

Baltzer M., Capiello L., Santis de R.A., Manganelli S. (2008), *Measuring Financial Integration in the New EU Member States*, ECB, Occasional Paper, no. 81.

Capiello L., Gerard B., Kadareja A., Manganelli S. (2006), *Equity Market Integration of New EU Member States*, in: *Financial Development, Integration and Stability. Evidence from Central, Eastern and South-Eastern Europe*, ed. K. Liebscher, J. Christl, P. Mooslechner, D. Ritzberger-Grünwald, E. Elgar, Chentelham, Northampton.

Czupryna M. (2011), *O współzależności giełd na przykładzie giełdy polskiej i niemieckiej*, Annales Universitatis Mariae Curie-Skłodowska. Sectio H. „Oeconomia" vol. 47, no. 3, pp. 109–118.

Engle R.F., Granger C.W.J. (1987), *Co-integration and error correction: Representation, estimation and testing*. "Econometrica" vol. 55, no. 2, pp. 251–276.

Fratzscher M. (2002), *Financial Market Integration in Europe: On the Effects of EMU on Stock Markets*, "International Journal of Finance & Economics" vol. 7, no. 3, pp. 165–193.

Geweke J., Porter-Hudak S. (1983), *The estimation and application of long memory time series models*, "Journal of Time Series Analysis" vol. 4, pp. 221–238.

Granger C.W.J. (1981), *Some Properties of Time Series Data and Their Use in Econometric Model Specification*, "Journal of Econometrics" vol. 16, no. 1, pp. 121–130.

Granger C.W.J., Joyeux R. (1980), *An introduction to long-memory time series models and fractional differencing*, "Journal of Time Series Analysis" vol. 1, pp. 15–29.

Johansen S. (1988), *Statistical Analysis of Cointegrating Vectors*, "Journal of Economic Dynamics and Control" vol. 12, pp. 231–254.

Johansen S., Juselius K. (1990), *Maximum Likelihood Estimation and Inference on Cointegration–with Applications to the Demand for Money*, Oxford Bulletin of Economics and Statistics, vol. 52, no. 2, pp. 169–210.
Mrzygłód U. (2011), *Procesy integracyjne na rynkach kapitałowych Unii Europejskiej*, Materiały i Studia Narodowego Banku Polskiego, no. 257.
Nielsen M.Ø., Shimotsu K. (2007), *Determining the cointegrating rank in nonstationary fractional systems by the exact local Whittle approach*, "Journal of Econometrics" vol. 141, pp. 574–596.
Robinson P.M., Marinucci D. (2001), *Narrow-band analysis of nonstationary processes*, "Annals of Statistics" vol. 29, no. 4, pp. 947–986.
Robinson P.M., Yajima Y. (2002), *Determination of cointegrating rank in fractional systems*, "Journal of Econometrics" vol. 106, no. 2, pp. 217–241.
Shimotsu K. (2010), *Exact local Whittle estimation of fractional integration with unknown mean and time trend*, "Econometric Theory" vol. 26, pp. 501–504.
Shimotsu K. (2012), *Exact local Whittle estimation of fractionally cointegrated systems*, "Journal of Econometrics" vol. 169, pp. 266–278.
Shimotsu K., Phillips P. (2005), *Exact local Whittle estimation of fractional integration*, "The Annals of Statistics" vol. 33, no. 4, pp. 1890–1933.
Yusupova E. (2005), *The Equity Market Integration of the Central and Eastern European Countries. Does the Timing of EMU Accession Matter?,* Kiel Institute for World Economics, Working Paper no. 429.

### ZALEŻNOŚCI DŁUGOOKRESOWE POMIĘDZY GIEŁDAMI W WIEDNIU, FRANKFURCIE I W WARSZAWIE

**Streszczenie:** W artykule zaprezentowano badanie występowania długookresowych zależności pomiędzy głównymi indeksami trzech europejskich rynków akcji: we Frankfurcie, Wiedniu i w Warszawie. Dwa pierwsze są rynkami rozwiniętymi, natomiast giełda w Warszawie jest ciągle postrzegana jako rynek rozwijający się. Na podstawie dziennych danych przeprowadzono analizę występowania klasycznej kointegracji $I(1)/I(0)$. Jej wyniki zostały porównane z wynikami uzyskanymi za pomocą analizy ułamkowej kointegracji.

**Słowa kluczowe:** kointegracja, ułamkowa kointegracja, rynek akcji, rynki wschodzące

## Citation

Wójtowicz T. (2105), *Long-Term Relationships between the Stock Exchanges in Frankfurt, Vienna and Warsaw*, Zeszyty Naukowe Uniwersytetu Szczecińskiego nr 855, „Finanse, Rynki Finansowe, Ubezpieczenia" nr 74, t. 1, Wydawnictwo Naukowe Uniwersytetu Szczecińskiego, Szczecin, s. 217–227; www.wneiz.pl/frfu.