THE IMPACT OF CHANGES OF CAPITAL COST IN ENTERPRISE VALUATION

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Introduction

The need to determine the value of the company is almost commonplace in today’s business. Companies are valued at least for the purpose of sale transactions, privatization, issue of new shares, mergers, acquisitions and divisions.1

Depending on the subject of valuation and its aim, in the literature on valuation and the practice of its preparation a wide variety of valuation methods is presented.2 Regardless of the aim of valuation, the impact of the external environment – the situation of the economic sector, company size, and availability of data, the majority of practitioners and theorists assume that the best and most universal way to approach the problem of company valuation is by means of the discount cash flow models.3 In order to use the income methodology to estimate the enterprise value it is essential to determine specially designed discount rates to calculate the present value of future cash flows.4 These rates are also called cost of capital, and can be explained as the expected rate of return on the committed capital.5

However, currently available theoretical models do not allow us to calculate these discounts with one hundred percent certainty. Difficulties in calculating discounts are the

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result of problems occurring in the determination of the cost of equity – the interest rate, which investors expect to gain in return for investment of their resources in the company. Cost of equity can be estimated by using qualitative methods (submission technique) and analytical methods.\(^6\) The most common method of determining the premium for equity commitment according to R.F. Bruner, K.M. Eades, R.S. Harris, and R.C. Higgins\(^7\) study is the Capital Asset Pricing Model\(^8\), which was indicated by 81% of CFOs and 80% of financial advisors surveyed. This model attempts to explain the capital asset pricing in the context of its risk.\(^9\) It has also been identified as the most common model (85% respondents) in the study conducted by J. Al.-Ali and T. Arkwright,\(^10\) which comprised 450 largest British corporations under the criteria of trading volume. Therefore, in this article, the analysis of the impact of the cost of equity for the company valuation will be carried out in accordance with the CAPM model.

**Valuation model**

The cost of the funding committed by the company is closely connected with the source of its origin. The required rate of return for investors for their funds transferred to the company is called the cost of equity.\(^11\) This cost is not homogeneous, because the donor of capital can get different kinds of profits for his contribution. Generally, in the case of a joint stock company there is the capital cost of the preferred stock and the cost of capital of ordinary shares. The funds received by the company from other sources are called debt. The structure of this financing can also be varied. The basic criteria for the allocation of debt is the horizon of the loan, purpose, form of transfer – loan, lease, bonds,\(^12\) etc., security – mortgage, bill of exchange, pledge and relationship with the financing and cost incurring companies. Whenever the company incurs a new debt it’s the cost of its transfer is determined.

The cost of a new debt incurred by the company is connected with the method of financing, the object and source of funding and the economic situation of the company. However, its amount is always determined uniquely, and it is the cost of obtaining the debt by capital. Given that the activities of each enterprise can be financed by both equity and


\(^8\) The Capital Asset Pricing Model (CAPM) has been developed independently by J. Lintner (1965), J. Mossin (1966) and W. Sharpe (1664).


\(^11\) H. Zadora: op.cit.

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debt in various ways, the overall cost of capital of the entity in a fixed time $0 < t < \infty$ can be determined by\textsuperscript{13}:

$$WACC_t = \sum_{i=1}^{n} w_i k_i (1 - T_t \cdot \Xi_{it})$$  \hspace{1cm} (1)\textsuperscript{14}

where:
- $w_i$ – share of the $i$-th source of capital in company funding at time $t$,
- $k_i$ – cost of the $i$-th source of funding at time $t$ without considering the possibility of using the tax shield,
- $T_t$ – income tax rate at the time $t$,
- $\Xi_{it}$ – it takes the value 1 if at time $t$ the fee for using the $i$-th source of capital can decrease the tax due, and 0 otherwise,
- $n$ – number of different ways of financing.

In order to estimate the value of the company in addition to determining the weighted average cost of capital\textsuperscript{15} it is also essential to determine the cash flow generated by the company in each period of the financial forecast. In WACC methodology the approach to financial flows should be built according to standard FCFF (Free Cash Flows for Firm)\textsuperscript{16}. Assuming that the company under valuation can operate indefinitely, its value before taking into account the balance sheet adjustments is determined by the following formula:

$$EV = \sum_{k=1}^{\infty} \frac{FCFF_k}{\prod_{i=1}^{k}(1 + WACC_i)}$$  \hspace{1cm} (2)

where:
- $FCFF_i$ – free cash flow for firm in the $i$-th period of forecast,
- $WACC_i$ – the weighted average cost of capital for the company – current year.

In practice, a series of numbers determined by formula (2) due to the fact that it sums smaller and smaller numbers can be approximated by the following formula:

$$EV \approx \sum_{k=1}^{n} \frac{FCFF_k}{\prod_{i=1}^{k}(1 + WACC_i)} + \frac{FCFF_n \cdot (1 + g)}{\prod_{i=1}^{n}(1 + WACC_i)} WACC_n - g$$  \hspace{1cm} (3)

where:
- $g$ – is projected rate of company growth after a period of detailed forecast.

\textsuperscript{14} The formula defines the cost of capital according to the average weighted cost of capital approach.
In formula (3) the expression \( \frac{FCFF_n \cdot (1 + g)}{(WACC_n - g)} \) is called the residual value of the firm under valuation,\(^{17}\) a \( \prod_{i=1}^{n} \left( 1 + WACC_i \right) \) (4) is a discount factor of the residual value to the present time. The use of the discount cash flows methodology in order to determine the value of enterprise according to theory requires assumptions about the company development. Therefore, because of this increase, the cash flow generated by the company in the last period of financial forecast \( FCFF_n \) will be almost always positive. Consequently, the higher the expected rate \( g \) the higher the level of the residual value and the final value of the company. Both practically and theoretically there is no clear and widely accepted approach to determine the value of \( g \). The theoretical approaches (which can be found for example in the publication written by Szablewski and Tuzimek\(^{18}\)) relate the magnitude of business growth after detailed forecasts with the level of inflation rate. The natural limitation form above for value \( g \) is \( WACC_n \). In practice, it is assumed that \( g < r_n \), where \( r_n \) is a projected rate of inflation in the last year of detailed forecast. This assumption is justified in an intuitive way, since no company, even the ideally managed one, will be able to increase the level of margins earned annually, which would occur if \( g > r_n \). However, still it is not decided which way is the best to determine the value of \( \varepsilon = r_n - g \) assuming that \( \varepsilon > 0 \).

**Functional analysis of enterprise value**

The value of the company before adjustments, determined by formula (3) in the process of analysing the impact of the weighted average cost of capital, can be treated as a function of \( n \)-variables, where \( n \) – is the numbers of periods in the financial forecast. Therefore, it is possible to define function \( F : (1, 2)^n \rightarrow R \) by the following formula:

\[
F(t_1, t_2, \ldots, t_n) = \sum_{k=1}^{n} \frac{c_k}{\prod_{i=1}^{k} t_i} + \frac{c_n (1 + g)}{\prod_{i=1}^{n} t_i \cdot (t_n - 1 - g)}
\]

where \( t_i = 1 + WACC_i \), \( c_i = FCFF_i \), for \( i = 1, 2, \ldots, n \). Naturally, after taking into consideration model assumptions it is obvious that there is at least one index \( p \in \{1, 2, \ldots, n\} \) such as \( c_p \neq 0 \).

If for each \( i = 1, 2, \ldots, n \) cash flows generated by the company \( c_i > 0 \), then function \( F \) decreases with respect to each variable. Therefore, in order to obtain the highest possible company valuation, the weighted average cost of capital in each year forecasts should be re-


\(^{18}\) A. Szablewski, R. Tuzimek: *op.cit.*, p. 4.
duced as much as possible (but methodologically acceptable\(^{19}\)). In the situation when FCFF generated by the company is not non-negative in each year of the financial forecast, the answer to the question of how the function of the company value depends on the individual variables of the cost of capital (WACC in different years) is not so obvious.

**Lemma:** Function \( F : (1, 2)^n \rightarrow \mathbb{R} \) defined by formula (5) does not have a local extreme.

**Proof:** Let us suppose that function \( F \) has a local extreme point \( t^* = (t^*_1, t^*_2, \ldots, t^*_n) \in (1, 2)^n \). Because function \( F \) is differentiable as a superposition of elementary functions, from a necessary condition for the occurrence of the extreme we have for \( i = 1, 2, \ldots, n \)

\[
\frac{\partial F}{\partial t_i}(t^*_1, t^*_2, \ldots, t^*_n) = 0.
\]

For function \( F \) the first-order partial derivatives are:

\[
\frac{\partial F}{\partial t_i}(t_1, \ldots, t_n) = \begin{cases} 
- \frac{1}{t_i} \sum_{k=1}^{n} c_k + \frac{c_n (1 + g)}{d_n (t_n - 1 - g)} & i \neq n \\
- \frac{c_n}{t_n d_n} \left( 1 + \frac{(1 + g) (2t_n - 1 - g)}{(t_n - 1 - g)^2} \right) & i = n
\end{cases}
\]

where \( d_k = \prod_{l=1}^{k} t_l \) for each \( k \in \{1, 2, \ldots, n\} \). In particular, again from a necessary condition for the occurrence of local extreme\(^{20}\) we have

\[
0 = \frac{\partial F}{\partial t_n}(t_1^*, t_2^*, \ldots, t_n^*) = - \frac{c_n}{t_n d_n} \left( 1 + \frac{(1 + g) (2t_n^* - 1 - g)}{(t_n^* - 1 - g)^2} \right).
\]

This equation is satisfied if and only if: \( c_n = 0 \) or \( 1 + \frac{(1 + g) (2t_n^* - 1 - g)}{(t_n^* - 1 - g)^2} = 0 \).

Hence \( 0 = \left( t_n^* - 1 - g \right)^2 + (1 + g) (2t_n^* - 1 - g) = \left( t_n^* \right)^2 \). Directly from the definition of \( t_n^* \) we have \( t_n^* > 0 \), which gives a contradiction. Accordingly \( c_n = 0 \). Substituting this value into the equation \( \frac{\partial F}{\partial t_{n-1}}(t_1^*, t_2^*, \ldots, t_n^*) = 0 \), we get \( c_{n-1} = 0 \). Following this procedure, we can infer that only when \( c_1 = c_2 = \ldots = c_n = 0 \) the first order partial derivatives of function \( F \) are equal zero, which is in contradiction to the assumption of no triviality of function \( F \). Therefore, the assumption that function \( F \) has extremes is false.

In a situation where not all of FCFF in the forecast are non-negative, function \( F \) can both decrease and increase the individual variables. From the methodological point of view in the discounted cash flows it is usually assumed that \( c_n > 0 \). Because of this assumption,
independently of other projected cash flows in the forecast, function \( F \) has the following property:

Property 1: For any fixed value \( c_1, \ldots, c_{n-1} \) function \( F \) given by formula (5) is decreasing with respect to variable \( t_n \).

In order to verify this property it is sufficient to note that \( \frac{\partial F}{\partial t_n}(t_1, \ldots, t_n) < 0 \). Indeed

\[
\frac{\partial F}{\partial t_n}(t_1, \ldots, t_n) = -\frac{c_n}{t_n d_n} \left( 1 + \frac{(1+g)(2t_n - 1 - g)}{(t_n - 1 - g)^2} \right)
\]

\[
= -\frac{c_n \left( t_n - 1 - g \right)^2 + (1+g)(2t_n - 1 - g)}{t_n d_n (t_n - 1 - g)^2} = -\frac{c_n}{t_n d_n} \frac{t_n^2}{(t_n - 1 - g)^2} < 0,
\]

cause \( c_n > 0 \) from the assumptions of model DCF, \( d_n > 0 \) as the product of positive numbers

\( 0 < t_n - 1 - g = 1 + WACC_{n - 1} - g = WACC_n - g \) again due to the model assumptions, since no company can grow indefinitely.

Property 2: Function \( F \) defined by formula (5) increases with respect to variable \( t_{n-1} \) if and only if \( c_{n-1} < 0 \) and \( t_n - g - 1 > -(c_n / c_{n-1}) \).

Proof: Function \( F \) is increased by variable \( t_{n-1} \) when the following condition \( \frac{\partial F}{\partial t_{n-1}}(t_1, \ldots, t_n) > 0 \) is satisfied. Directly from the definition of the partial derivative of the first order we have:

\[
\frac{\partial F}{\partial t_{n-1}}(t_1, \ldots, t_n) = -\sum_{k=n-1}^{n} \frac{c_k}{t_{n-1} d_k} - \frac{c_n (1+g)}{t_{n-1} d_n (t_n - 1 - g)}
\]

\[
= -\frac{1}{t_{n-1} d_n} \left( c_{n-1} \cdot t_n + c_n + \frac{c_n (1+g)}{(t_n - 1 - g)} \right).
\]

This expression is positive if and only if \( c_{n-1} \cdot t_n + c_n + \frac{c_n (1+g)}{(t_n - 1 - g)} > 0 \), which in turn takes place if and only if \( c_{n-1} (t_n - g - 1) + c_n < 0 \). Let us suppose now that \( c_{n-1} > 0 \). Then \( t_n - 1 - g < -(c_n / c_{n-1}) < 0 \), which is contrary due to the fact that \( t_n - 1 - g = 1 + WACC_n - 1 - g = WACC_n - g > 0 \). Hence we have \( c_{n-1} < 0 \) and \( t_n - g - 1 > -(c_n / c_{n-1}) \).

Identification of the extent of the relationship between fixed \( c_1, \ldots, c_n \) and restrictions on individual variables \( t_1, t_2, \ldots, t_n \), for which the function increases for subsequent variables \( t_1, t_2, \ldots, t_{n-2} \) should be done in a recursive. Function \( F \) is increased by variable \( t_{n-2} \) if and only if \( (t_n - 1 - g)(c_{n-2} t_{n-1} + c_{n-1}) + c_n < 0 \). Thus, the value of the company grown with the increase in weighted average cost of capital in year \( n - 2 \) is not sufficient to \( c_{n-2} < 0 \) This follows from the fact that the increase in the discount reduces the level of negative cash flow during the detailed forecast, but at the same time causes a reduction of the positive
flow of the subsequent years and the residual value in the end.\textsuperscript{21} Due to the complexity of the analysis the variability of the cases for the following variables for function $F$ is not presented here.

**Determination of the level of change in the valuation**

The value of the company before balance sheet adjustments, determined by formula (3) depends on the $2n + 1$ variables. Analyzing the impact of changes $WACC_1$, $WACC_2$, …, $WACC_n$ on enterprise valuation, variables $c_1$, $c_2$, …, $c_n$ and $g$ will be interpreted as parameters. The values of these parameters will be determined in advance for each study.

An overview of business valuation reports prepared by 23 major brokerage houses operating in Poland in the years 2001–2012 shows that in the study of the impact of changes on average cost of equity valuation levels, special attention should be paid to the forecast of cash flow in the $n$–th period and projected growth after the detailed forecast. In the case of 230 brokerage house recommendations of the value of more than 138 companies from various industries, the average of sharing the present value of the residual value is 58.17% in the total valuation of the company. The quartile of the first row in the sample is 46.80%, the median is 54.48%, and the third row quartile is 68.46%. The smallest share of the residual value is 18.90%. The biggest in turn is 142.73%.

Turning to the analysis of changes in the value of the company if in the FCFF valuation model there is an increased growth rate after the forecast from level $g$ to $g + a$, where $a \in \mathbb{R}$ is such as $g + a < WACC_n$, the changes of enterprise value ($CEV$) will be defined by the following formula:

$$CEV = \frac{DRV}{EV} \cdot \frac{a \cdot t_n}{(1 + g)\left(t_n - 1 - g - a\right)} \quad (6)$$

where:

$CEV$ – the changes of enterprise value ($EV$),

$DRV$ – discounted (present) value of the residual.

Equation (6) is a simplified form of $(EV_a - EV) / EV$, where $EV_a$ is the enterprise value determined for the growth rate after the financial forecast of $g + a$.

If there are changes in the amount of the weighted average cost of capital in the following years, starting with a fixed period $i_0$–th year, by the value $1 + a_i$ respectively for $i \in \{i_0, \ldots, n\}$, where $a_i$ are limited such as $t_i (1 + a_i) > 0$ for $i \in \{i_0, \ldots, n - 1\}$ and $t_n (1 + a_i) - g > 0$, then the change in the company’s value is given by the formula:

\textsuperscript{21} T. Wiśniewski: *Błędy szacowania kosztu kapitału w decyzjach inwestycyjnych i ich skutki*. „Badania operacyjne i decyzje” 2008, No. 3.
CEV = \(-\frac{1}{EV} \left\{ \sum_{k=i}^{n} \frac{c_k \prod_{l=i}^{k} a_l}{d_k \prod_{l=i}^{k} (1 + a_l)} \right\} + \frac{a_n c_n (1+g)}{d_n \prod_{l=i}^{n} (1 + a_l) t_n - 1 - g} \) \tag{7}

A derivation of equation (7) is simplified arithmetic expression \( \frac{EV \Pi a - EV}{EV} \), where \( EV_{\text{initial}} \) is the value of the company before adjustments for new discount factors.

Conclusions

In this article, the theoretical criteria are described which explain how the average cost of capital affects the outcome of the valuation of the company. Additionally, the relationship between the change in value \( WACC_1, \ldots, WACC_n \) and \( FCFF_1, \ldots, FCFF_n \) has been determined in order to verify that the discount rate in the valuation model has not been selected to unduly influence the level of valuation\(^{22}\). Therefore, we have a tool which at the initial stage of the analysis indicates the discount pricing models, which allows for the manipulation of the value of the company through appropriate decrease or increase of the value of \( t_1, \ldots, t_n \). In order to make this assessment model plausible it is necessary minimize the risk of making decisions based on incorrect pricing\(^{23}\).

This paper also sets patterns determining the percentage change in the value of the company based on value \( t_1, \ldots, t_n \) and \( g \), which should definitely make it easier to carry out a sensitivity analysis of changes in the value of the company. As demonstrated by empirical research reports from brokerage valuations special emphasis should be placed on the determination of the residual value of dual companies. Further studies should be conducted in the case of FCFF valuation in two areas: errors in models assumptions and calculation methods of the weighted average cost of capital.

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\(^{22}\) B. Miedziński: Nadużycia w wycenie Przedsiębiorstw, Oficyna Wydawniczo-Poligraficzna „Adam” 2012, p. 50.

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**Summary**

The purpose of this article is to provide analytical tools allowing for initial identification of sensitive factors in the discount cash flow model. There are boundary conditions that influence the level
of valuation by changing the discount rate. The study also derived formulas to determine changes in the enterprise, depending on the assumed level of the weighted average cost of capital and the growth rate after the financial forecast. The results provide important insight for practitioners of business, enabling them to minimize the risk of partial business decisions based on incorrect pricing. They also help speed up the examination of the sensitivity of the valuation of the company in the assumptions about discount factors and the growth rate of the forecast period.

WPŁYW ZMIANY KOSZTU KAPITAŁU WŁASNEGO NA WYCENĘ PRZEDSIĘBiorSTWA

Streszczenie

Celem niniejszego artykułu jest przedstawienie narzędzi analitycznych pozwalających na wstępną identyfikację wrażliwych elementów w modelach wyceny metodą dochodową na zmianę wartości czynników dyskontujących. Wyznaczone zostają warunki brzegowe, które umożliwiają wpływanie na poziom wyceny poprzez zmianę stóp dyskontowych. W pracy wyprowadzono również wzory na wyznaczenie zmiany wartości przedsiębiorstwa w zależności od przyjętego poziomy średnioróżowego kosztu kapitału oraz stopy wzrostu po okresie prognozy. Otrzymane wyniki mają szczególnie znaczenie dla praktyków działalności gospodarczej, umożliwiając im częściowe minimalizowanie ryzyka podejmowania decyzji biznesowych na podstawie nieprawidłowych wycen. Pozwalają również zdecydowanie przyśpieszyć przeprowadzanie analizy wrażliwości wyceny przedsiębiorstwa w zakresie założeń dotyczących czynników dyskontujących oraz stopy wzrostu po okresie prognozy.