THOMAS HERING
CHRISTIAN TOLL
POLINA K. KIRILOVA

DECISION-ORIENTED BUSINESS VALUATION
IN PREPARATION FOR A COMPANY PURCHASE

Keywords: investment decisions, buyer value, linear models

Słowa kluczowe: decyzje inwestycyjne, wartość dla kupującego, modele liniowe

JEL classification: D43, G11, C32

Introduction

A business valuation is based on the assessment of the future uncertain cash stream flowing between the company and its owners. For example, a purchase is not disadvantageous as long as the price paid in exchange for the obtained object (valuation object) does not exceed its value for the acquirer (valuation subject). The price expresses the negotiation outcome, while the value – according to the subjective value theory founded by Hermann Heinrich Gossen and Carl Menger\(^1\) – results from its marginal utility regarding a predefined subjective aim. In consequence, the valuation process depends on the target function (usually prosperity maximization, i.e. wealth or income maximization) as well as on the decision field, which is constituted of all opportunities for action available to the valuation subject.

A company should be purchased only if the purchase results in a target achievement level which is at least equivalent to that attainable when the transaction is not concluded. For this reason, a business valuation should help to judge about the economic adequacy of a given price for the transfer of the valuation object. It is important to bear in mind that each appraisal is subject to the intended purpose. The functional business valuation theory facilitates such purpose-orientated valuation by providing guidelines for the different valuation tasks. The three main functions, which imply an intended change in ownership, are

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decision, mediation and argumentation functions. The most important one among them, the decision function, provides the decision value for the valuation subject as the limit of the subject’s will to concede in the specific conflict situation. In the case of a company acquisition, the seller gives up ownership of the company to the purchaser and in return receives compensation, typically monetary. The focal point of the emerging negotiations is the agreement which stipulates the conditions of the ownership transfer. To avoid any economic disadvantage, the presumptive buyer should be aware of his individual limit of concession willingness in the given negotiations. Since it is the amount of monetary compensation which is the only controversial issue, this limit is equal to the maximum price that the presumptive buyer will be able to pay, without deteriorating its position compared to that held in the case when a company is not acquired (marginal or critical price).

The models which have to be developed based on investment theory in order to valuate a company can be either general or partial. According to the Fisher separation theorem in a fictitious perfect capital market the marginal price can be obtained as a future earnings value applying a partial model, i.e. without the necessity to take into account the entire decision field of the valuation subject. Then, the interest rate is exogenous. In a real imperfect market it is inevitable to consider interdependent investment, financing and consumption decisions simultaneously. The consumption preference of the valuation subject is expressed in the predefined structure of withdrawals and is no longer separable from the time preference of the money. It affects the temporal distribution and the level of the individual withdrawals as well as the investment and financing decisions. Thus, the shadow prices for each period (the endogenous marginal interest rates), which are required for the partial model, can only be determined for the specific conflict situation as a byproduct of the general model solution. Subsequently we will compute the decision value in preparation for a company purchase under realistic conditions using the general “state marginal price model”. For the sake of simplicity, all modeling is done under the assumption of certainty.


Description of the General Model – State Marginal Price Model

In order to calculate the decision value in the context of a company purchase, the state marginal price model will be introduced below. This model combines the advantages of the mixed integer model of Laux/Franke with the two-step procedure of Jaensch and Matschke. Laux/Franke calculate the marginal price of a certain cash stream within an imperfect capital market by applying the multi-period simultaneous planning approaches of Hax und Weingartner. Thereby, they set an obviously advantageous price into their linear optimization model. Afterwards they vary this price continuously in a parametric manner until the change in ownership of the valuation object becomes disadvantageous. This means that the variable for the company purchase is no longer part of the optimal investment and financing program. So the model of Laux/Franke requires a numerically extensive mixed-integer parametric optimization. The models of Jaensch and Matschke handle this problem by determining the decision value in a two-step procedure. The first step is to calculate – as a so-called base program – the investment and financing program which maximizes the target function value (income EN or asset value GW) under unchanged property conditions regarding the valuation object. Subsequently, in a second step the valuation object has to be integrated into the investment program of the presumptive buyer in the case of a company acquisition. Then, the maximum affordable price as a down payment is searched. Hence, the decision field is extended by adding the valuation object at a price of p and additionally it is supplemented by the condition that at least the target function contribution of the base program must be achieved again. The result of this second step is the so-called valuation program with its optimal value p* that indicates the requested upper price limit as a down payment (i.e. decision value or marginal price). As opposed to Laux/Franke, the models of Jaensch and Matschke has the drawback of not considering the imperfect capital market over the passage of time. Instead, a single accumulated number of success is assigned to each multi-period investment and financing object. The state marginal price model combines the advantages of these models in a way that permits determination of the marginal price at the time t = 0 under imperfect capital market conditions by setting up a base and a valuation approach without being dependent on the mixed-integer parametrical optimiza-

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tion as the Laux/Franke model. In the following sections it is assumed that the valuation subject pursues the target income maximization, wherefore he strives for the greatest possible size \( EN \) of a structured withdrawal stream.\(^{13}\) The actual amount of the desired withdrawal at time \( t \) then results from the intended temporal structure that is predetermined by the consumption preference. Thus, the size \( EN \) that has to be maximized gets converted into the actually desired withdrawals \( \bar{w}_t \cdot EN \) with the help of the weightings \( \bar{w}_t \), which reflect the consumption preference. This results in a stream of withdrawals of the intended temporal structure. To ensure the existence of the company beyond the planning horizon \( n \), the autonomous cash flow \( b_t = b_n \) has to additionally consider a sufficient terminal asset as a fictive withdrawal. This terminal asset, as the present value of a perpetual annuity, permits the continuation of the desired dividend level. The autonomous cash flow \( b_t \), which also results from the other predetermined payments (e.g., from current business operations and existing loan obligations), is independent of the assessed available objects \( j \) and can be positive, negative or zero. Alternatively, the substance of the company could be protected by considering the fictive withdrawal at the planning horizon using an adequately high weighting \( \bar{w}_n \).

Furthermore, for the valuation subject as the presumptive buyer, the following assumptions are made: The planning period extends \( n \) periods, whereas \( t = 0 \) defines the valuation and decision point in time. In the baseline situation \( m \) investment and financing objects \( j \) are available for the buyer \((j = 1, ..., m)\). This also includes at any point in time the opportunity of borrowing money, the opportunity to invest money in financial assets as well as an unlimited cash management that is defined by the payment stream \((-1, 1)\). The cash stream of the object \( j \) is determined as follows: \( g_j := (g_{j0}, g_{j1}, ..., g_{jt}, ..., g_{jn}) \) with \( g_{jt} \) being the cash flow of object \( j \) at time \( t \). How often an investment or financing object \( j \) can be realized is shown by the variable \( x_j \). For the variables \( x_j \) there are upper bounds \( x_j^{\text{max}} \) (which may also be \( \infty \)). The \( n + 1 \) liquidity constraints ensure that at any point in time \( t \) the sum of all cash flows remains non-negative. In other words, the liquidity constraints have to ensure that at any time \( t \), the sum of all cash flows from the realized investment and financing objects as well as from autonomous payments are sufficient to permit the desired withdrawal \( \bar{w}_t \cdot EN \) Moreover, the variables \( EN \) and \( x_j \) are also limited to non-negative values.

All in all, the base program (the combination of financing and investment options that maximizes the buyer’s success without the acquisition of the company in question) results from the linear optimization approach “max \( \text{Ent}_n \)”\(^{14}\) presented in figure 1. The simplex


algorithm\textsuperscript{15} calculates the optimal solution of this linear approach resulting in a maximum target function value $EN^*$. Buying the firm at a price $p$ is then only economically viable if the valuation program at least yields the optimal target function value $EN^*$ of the base program.\textsuperscript{16} Thus, the approach to determine the valuation program contains the restriction $EN \geq EN^*$. If the buyer takes over the company, he receives its cash stream $g_K = (0, g_{K1}, g_{K2}, \ldots, g_{Kt}, \ldots, g_{Kn})$. For this reason, these cash flows $g_{Ki}$ have to be added to the autonomous payments $b_t$. In exchange he pays the price $p$ at time $t = 0$. The decision value must then be determined.\textsuperscript{17} Therefore, the presumptive buyer has to answer the question which price he can just afford to pay, without the acquisition putting him into a worse position than if, instead of acquiring the company, he had implemented the available base program. In this manner, $p$ must consequently be maximized, taking into account the restrictions of the original decision field, extended by the payment stream from the purchased company and subject to the additional condition of not violating $EN^*$. The answer can be found with the help of the valuation approach “max $U$” in figure 1.\textsuperscript{18} Again, the simplex algorithm generates the optimal solution (valuation program) and thus provides not only the marginal price $p^*$ (max. $p$, i.e. the decision value) but also the buyer’s optimal investment and financing program, restructured by the inclusion of the acquired company’s payments in exchange for the purchase price $p = p^*$.

**Exemplary Presentation of the General Model – State Marginal Price Model**

Now, a fictive example will be conceived in order to illustrate the procedure presented above. Let us imagine firm A aspiring to purchase company K. The management forecasts that the acquisition of company K will be accompanied in the planning period ($n = 5$) by cash stream $g_K = (0, 20, 25, 30, 20, 10)$ and from the sixth year on by a perpetual annuity in the amount of 5 monetary units (MU). At the valuation date $t = 0$ company A expects that the previous business activity leads to a perpetually arising deposit excess from internal financing $b_t$ (i.e. autonomous cash flow) amounting to 100 MU. In order to reduce the


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complexity of the example, we assume that firm A has only a few investment and finance options. Firstly, at \( t = 0 \) company A can invest in a tangible asset (e.g., modernization of the existing production lines) which is associated with the payment stream \((-160, 15, 15, 15, 15, 315)\). Secondly, firm A is able to invest an unlimited amount of money in financial assets that promise a return of 5% per annum (p.a.). A five-year annuity loan is available at \( t = 0 \) provided by the local bank at an annual interest rate of 7% restricted to 50 MU to finance the acquisition. Furthermore, company A can debit a revolving line of credit at a short-term interest rate of 10% p.a. limited to 80 MU. Company A pursues the target \textit{income maximization}, seeking for a uniform income stream which has to be perpetuated at the planning horizon. To achieve this, the last withdrawal \( \bar{w}_n \cdot EN \) must contain not only the normal amount \( EN \) but also the present value of the perpetual annuity. Based on a generally estimated interest rate of 5% p.a. for \( t > n = 5 \), the intended temporal structure of the weightings is \( \bar{w}_i = 1 \) for \( 0 < t < 5 \) and \( \bar{w}_n = 21 \).

Taking the given decision field (without the acquisition of company A = baseline situation), a uniform income stream of size \( EN^* = 111,5312 \) MU results from the \textit{base approach}. At the end of the planning horizon a deposit in the amount of 2,230,6236 MU is accrued, which gives – at a rate of 5% p.a. – the intended perpetual annuity \( EN^* \) that is intended from the sixth year on. Hence, withdrawals in the amount of 111,5312 MU p.a. can be executed for all time. The tangible asset investment is realized completely. Not only is the annuity loan fully utilized, but also every year additional short-term financing is required. Consequently there will be no investments in financial assets. Table 1 shows the \textit{base program} as a complete finance schedule.\(^{19}\)

\[\begin{array}{c}
\text{max. } \text{Ent}_n; \quad \text{Ent}_n := EN = \sum_{j=1}^{m} g_{j0} \cdot x_j \leq b_0 \\
\sum_{j=1}^{m} g_{jt} \cdot x_j + \bar{w}_t \cdot EN \leq b_t \\
x_j \leq x_j^{\max} \\
x_j, EN \geq 0 \\
\end{array}\]

\[\begin{array}{c}
\text{max. } U; \quad U := p = \sum_{j=1}^{m} g_{j0} \cdot x_j + p \leq b_0 \\
\sum_{j=1}^{m} g_{jt} \cdot x_j + \bar{w}_t \cdot EN \leq b_t + g_{kt} \\
-EN \leq -EN^* \\
x_j \leq x_j^{\max} \\
x_j, EN, p \geq 0 \\
\end{array}\]

\(\forall t \in \{1, 2, ..., n\}\)

\(\forall j \in \{1, 2, ..., m\}\)

Figure 1. Base and valuation approach

\(^{19}\) The perpetual payment surplus from internal financing is taken into account in the example, using the generally estimated interest rate of 5% p.a. for \( t > n = 5 \).
Decision-oriented Business Valuation in Preparation for a Company Purchase

Table 1

<table>
<thead>
<tr>
<th>Time</th>
<th>t = 0</th>
<th>t = 1</th>
<th>t = 2</th>
<th>t = 3</th>
<th>t = 4</th>
<th>t = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_t$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>2.100</td>
</tr>
<tr>
<td>Tangible asset</td>
<td>–160</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>315</td>
</tr>
<tr>
<td>Annuity loan</td>
<td>50</td>
<td>–12,1945</td>
<td>–12,1945</td>
<td>–12,1945</td>
<td>–12,1945</td>
<td>–12,1945</td>
</tr>
<tr>
<td>Revolving line</td>
<td>10</td>
<td>19,7257</td>
<td>30,4240</td>
<td>42,1921</td>
<td>55,1370</td>
<td></td>
</tr>
<tr>
<td>Repayment</td>
<td>–11,0000</td>
<td>–21,6983</td>
<td>–33,4664</td>
<td>–46,4113</td>
<td>–60,6507</td>
<td></td>
</tr>
</tbody>
</table>

In a second step company K accompanied by the cash stream $g_k$ has to be integrated into the investment program of firm A. In exchange the presumptive buyer A has to answer the question which down payment he can afford to make without violating the size of the uniform income stream $E_{N^*}$. According to the valuation approach, this marginal price $p^*$ is 117,8531 MU. The complete valuation program can be described as follows: Company K is finally included in the optimal investment and financing program. Although the tangible asset investment can only be executed at 69,39065% due to the credit bottleneck in the second year, company A is still able to provide the uniform income stream of size $E_{N^*} = 111,5312$ MU. Hence, the desired dividends of the base program can also be realized in the valuation program. In addition to the annuity loan, the valuation program further debits the complete revolving line of credit amounting to 80 MU at time $t = 1$. In the other years, just as in the base program, no investments in financial assets are possible as borrowings have to be engaged in each planning period. Table 2 shows the valuation program as a complete finance schedule.

Table 2

<table>
<thead>
<tr>
<th>Time</th>
<th>t = 0</th>
<th>t = 1</th>
<th>t = 2</th>
<th>t = 3</th>
<th>t = 4</th>
<th>t = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_t + g_k$</td>
<td>100</td>
<td>120</td>
<td>125</td>
<td>130</td>
<td>120</td>
<td>2.210</td>
</tr>
<tr>
<td>Marginal price $p^*$</td>
<td>–117,8531</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tangible asset (69,3391%)</td>
<td>–110,9425</td>
<td>10,4009</td>
<td>10,4009</td>
<td>10,4009</td>
<td>10,4009</td>
<td>218,4181</td>
</tr>
<tr>
<td>Annuity loan</td>
<td>50</td>
<td>–12,1945</td>
<td>–12,1945</td>
<td>–12,1945</td>
<td>–12,1945</td>
<td>–12,1945</td>
</tr>
<tr>
<td>Revolving line</td>
<td>78,7956</td>
<td>80,0000</td>
<td>76,3249</td>
<td>67,2822</td>
<td>67,3353</td>
<td></td>
</tr>
<tr>
<td>Credit balance</td>
<td>–78,7956</td>
<td>–80,0000</td>
<td>–76,3249</td>
<td>–67,2822</td>
<td>–67,3353</td>
<td>2.230,6236</td>
</tr>
</tbody>
</table>

20 The perpetual payment surplus from internal financing and company K is taken into account in the example, using the generally estimated interest rate of 5% p.a. for $t > n = 5$. 

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In order to illustrate in what way changes in the decision field of the valuation subject may affect the decision value, the example will now be modified as follows: In addition to the annuity loan, the local bank grants an unlimited overdraft facility at a short-term interest rate of 10% p.a. Since the changes in the decision field of company A do not influence the optimal decisions made in the baseline situation, the base program shown in table 1 remains valid. Due to the improved financing situation, the maximum affordable price for \( K \) in the valuation program is now 143,3440 MU at time \( t = 0 \). Compared to the previous situation (i.e. credit limit), the tangible asset investment can now be completely realized in the valuation program. Just as the base program, the valuation program exhausts the annuity loan, requires borrowings every year and includes no investment in financial assets. Table 3 presents these results.

**Table 3**

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_t + g_{kt} )</td>
<td>100</td>
<td>120</td>
<td>125</td>
<td>130</td>
<td>120</td>
<td>2,210</td>
</tr>
<tr>
<td>Marginal price ( p^* )</td>
<td>-143,3440</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tangible asset</td>
<td>-160</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>315</td>
</tr>
<tr>
<td>Annuity loan</td>
<td>50</td>
<td>-12,1945</td>
<td>-12,1945</td>
<td>-12,1945</td>
<td>-12,1945</td>
<td>-12,1945</td>
</tr>
<tr>
<td>Revolving line</td>
<td>153,3440</td>
<td>157,4041</td>
<td>156,8703</td>
<td>151,2830</td>
<td>155,1370</td>
<td></td>
</tr>
<tr>
<td>Repayment</td>
<td>-168,6784</td>
<td>-173,1446</td>
<td>-172,5573</td>
<td>-166,4113</td>
<td>-170,6507</td>
<td></td>
</tr>
<tr>
<td>Withdrawal</td>
<td>-111,5312</td>
<td>-111,5312</td>
<td>-111,5312</td>
<td>-111,5312</td>
<td>-111,5312</td>
<td></td>
</tr>
<tr>
<td>Credit balance</td>
<td>-153,3440</td>
<td>-157,4041</td>
<td>-156,8703</td>
<td>-151,2830</td>
<td>-155,1370</td>
<td>2.230,6236</td>
</tr>
</tbody>
</table>

**Summary and Critical Appraisal**

The discussion above demonstrates that a company valuation cannot be executed in complete detachment from the individual expectations and planning of the specific valuation subject.\(^{21}\) Appraisal always depends on the subjective aim and the decision field of the valuation subject. Even when the same company is being assessed from the perspective of different valuation subjects the decision value may vary. The example shows that even the same valuation subject may come to diverging limits of concession willingness regarding the same valuation object when the underlying decision field changes.\(^{22}\) As a result, the maximum affordable price in exchange for the very same cash stream depends on the available opportunities for action.

Valuation methods based on the financing theory assume a fictitious perfect market. These methods do not take into consideration the individual expectations and planning of

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\(^{22}\) **Ibidem**, p. 368.
the specific valuation subject. Instead, they seek the one “true” value that has to be valid in general for everybody. For this reason, such methods are not appropriate to determine the decision value under realistic market conditions. Accordingly, its calculation can only be achieved by a business valuation based on investment theory, which can consider – as shown in this article – the existing market imperfections as well as the individual expectations and planning of the valuation subject. Of course, the determination of the decision value using the state marginal price model has also engendered criticism. In a general model all investment and financing objects are directly included in a simultaneous optimization approach. As this requires elaborate information gathering and processing, a centralized simultaneous planning with general models is often marked by complexity and clumsiness. Due to this, this procedure is usually not an option in a large-scale enterprise. So, even the most prudent and forward-looking analyst can neither capture all complex intra- and intercorporate circumstances nor generate absolutely reliable forecasts. There is no and there will never be a general model that is able to reflect all operational activities in a satisfying manner, because of the limited information gathering and processing capacity. Even if it were possible to develop a general model considering all data and interdependences, this model would suffer from a solution defect, since the optimal solution could not be found at economically viable expense. Moreover, a centralized simultaneous planning with general models is rather demotivating for operating units (divisions) subordinated to the administrative management (head office), because all decisions are made at management level. While the operating units as recipients of orders just submit information upwards, the decision makers are hopelessly overburdened. Therefore it is recommended to decompose the general model into several simpler partial models. For this purpose, the upper management level has to delegate certain decision-making power to the subordinate levels, which then can make decisions based on partial models. To ensure the integrity of planning, a link to the general model is still necessary, whereby the theoretical relations between general and partial models have to turn to account.


References


Prof. Dr. Thomas Hering, Dr. Christian Toll  
Dipl.-Kffr. Polina K. Kirilova  
Fern-Universität in Hagen, Germany

**Summary**

The aim of this paper is to show how the determination of an investment theory based decision value in preparation for a company purchase should be done. Thereby, the valuation subject (e.g., the buyer) acts in a real imperfect market. As far as the negotiation only considers the monetary compensation, from the presumptive buyer’s point of view the question arises which down payment he can afford without being put into a worse position than in the case of not acquiring the company. The state marginal price-model has been proved suitable to answer this question, because it can handle the specific circumstances of the particular buyer as well as existing market imperfections.

**DECYZYJNY PROCES WYCENY PRZEDSIĘBIORSTWA W PRZYGOTOWANIACH DO JEGO ZAKUPU**

**Streszczenie**

Celem niniejszego artykułu jest pokazanie, jak powinno się określać, w oparciu o teorię inwestycji, wartość decyzji, w celu przygotowania się do zakupu przedsiębiorstwa. Podmiot wyceny (np. kupujący) działa na rzeczywistym, niedoskonałym rynku. Jeżeli negocjacje dotyczą tylko rekompensaty pieniężnej, to z perspektywy kupującego pojawia się pytanie, jak wysoką cenę może zapłacić, zanim znajdzie się w gorszej sytuacji niż w przypadku nie nabycia spółki. Udowodniono, że model stanu i ceny granicznej nadaje się do odpowiedzi na to pytanie, ponieważ bierze pod uwagę sytuację konkretnego nabywcy oraz istniejące niedoskonałości rynku.